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WITH APPLICATION TO FISSION

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Adiabaticity Criterion for Charge Equilibration
with Application to Fission

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Abstract: The dispersion of a quantal oscillator with a time-dependent inertial mass is considered. For a special class of time dependence, an empirical method is formulated for predicting the asymptotic behavior of such a system. This method is then applied to the prediction of charge widths in strongly damped nuclear collisions and in fission.

In nuclear collisions and in fission the fluctuations in the collective coordinate associated with the giant dipole resonance (GDR) manifest themselves in the charge dispersion of the separating fragments [1-4]. As a neck begins to form between the nascent fragments the zero-point energy decreases and the period increases for GDR-like motion that involves the bulk flow of the neutron and proton fluids back and forth against each other. This increase in period is associated with the increase in the inertial mass caused by the increasingly restricted flow between the two halves of the system. When the neck size begins to decrease rapidly just prior to separation the collective motion is frozen in because it is no longer able to follow the change in shape.

In this note, we examine the expression governing the dispersion of an oscillator when the inertial mass changes with time. Whether or not the system is able to follow its adiabatic time development is found to depend on a critical quantity C . This enables us to formulate a simple "adiabaticity" criterion [4]. For a particular type of time dependence (of interest in nuclear processes) the quantity C is then used to establish an empirical procedure for determining the freeze out properties of a collective coordinate without having to resort to numerical solution of the time-dependent Schrodinger equation.

Once this empirical procedure is available we apply it to fission. In ref. [5] there are two different shape sequences

proposed for connecting the saddle and scission points in fission. They are difficult to distinguish because they both result in the same asymptotic kinetic energy release. Since one of these sequences is slow and the other rapid, we had hoped that the predicted charge dispersions would differ sufficiently to favor one or the other. Unfortunately, the process becomes nonadiabatic very close to scission where the time development of the two different shape sequences is nearly identical.

The equation of motion for the width of an oscillator with a time-dependent mass is

$$\ddot{\sigma} - \frac{\dot{b}}{b} \dot{\sigma} + bk \sigma = \frac{Ub^2}{\sigma^3} \quad (1)$$

where $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$.

The time-dependent quantity b is the inverse of the inertial mass associated with the collective coordinate, and k is the stiffness of the restoring force. The corresponding Hamiltonian is $H(t) = \frac{1}{2}b\dot{p}^2 + \frac{1}{2}kx^2$. The quantity U appearing in eq. (1) is a dynamical invariant whose value is $\hbar^2/4$ when the system starts out in its adiabatic ground state. (For more details see ref. [6].)

If σ is replaced in eq. (1) by $\sigma = z/\sqrt{b}$ the following expression is obtained:

$$\ddot{z} + \left[bk - \frac{3}{4} \left(\frac{\dot{b}}{b} \right)^2 + \frac{1}{2} \frac{\ddot{b}}{b} \right] z = \frac{U}{z^3} \quad (2)$$

This expression has the same form as the equation of motion for a particle in a time-dependent potential $V = 1/2(Kz^2 + U/z^2)$

where $K = [bk - 3/4(\dot{b}/b)^2 + 1/2(\ddot{b}/b)]$. For a static oscillator ($\dot{b} = 0$) the width (if perturbed) will oscillate about its adiabatic value $\sigma_{\text{adiabatic}} = (Ub/k)^{1/4}$ with the frequency 2ω , where $\omega = \sqrt{bk}$ is the classical oscillator frequency [7].

In the more general case of a time-dependent inertial mass, the stability of the width with respect to deviations from the adiabatic value depends on the sign of the stiffness K defined above. For the cases of interest to us we use this fact to construct a dimensionless adiabaticity criterion, which is used to establish an empirical scheme for estimating the freeze out width from the value of b and \dot{b} .

If we divide K by ω^2 and specialize to the case where $\ddot{b} = 0$ (and change the sign for aesthetic reasons) we obtain the quantity

$$C = \frac{3}{4} \frac{(\dot{b}/b)^2}{\omega^2} - 1 \quad . \quad (3)$$

This quantity can be put to use by first considering some idealized situations.

During the final stages of fission and strongly damped nuclear collisions the value of \dot{b} is nearly constant [1], and in fig. 1 we consider four different values (-0.002, -0.008, -0.012 and -0.020 (MeV dsec³)⁻¹) that cover the range of interest. (1 dsec = 10⁻²² sec.) We also set $k = 4$ MeV which is a typical value. The actual quantity plotted (as a solid straight line) in the upper part of the figure is the fourth power of Γ_0 , the adiabatic value of the full width at half

maximum, which comes from the expression $\Gamma_O^4 = 30.749(Ub/k)$.

(The numerical factor arises from the conversion between Γ and σ which is $\Gamma = 2\sqrt{2\ln 2} \sigma$ for a gaussian distribution.)

The dashed lines represent the actual values of Γ^4 obtained from a numerical solution of eq. (1). The width at large negative times follows the adiabatic value. As the system approaches scission at $t = 0$ the width is frozen in. The final value is larger for larger values of b^0 .

In the bottom half of the figure the quantity C is plotted for the same four cases. The values are all close to -1 for large negative times and then pass through zero and rise steeply as the system becomes nonadiabatic near the end point. To see if the quantity C could be employed in a prescription for estimating the freeze out value of Γ (call it Γ_f) we extended Γ_f back in time (see the light lines in the upper part of the figure) to the point where $\Gamma_O = \Gamma_f$. If we extend this critical time (t_c) downward to the lower half of the figure we can determine the corresponding critical value of the adiabaticity parameter (C_c). The fact that C_c is nearly equal to 1.108 for the whole range of interesting cases provides us with an empirical scheme for estimating the final width without the necessity of solving eq. (1).

One needs merely to calculate the value of C as a function of time. When the value reaches $C_c = 1.108$ the corresponding value of the adiabatic width Γ_O provides an excellent estimate of what the final freeze out value will be for Γ .

This procedure contains no provision for damping of the collective motion. In order to obtain an estimate of its importance we compared our calculation with a previous result that includes damping [1]. For the heavy-ion reaction $^{86}\text{Kr} + ^{92}\text{Mo}$ at $E_{\text{lab}} = 430$ MeV and for an impact parameter corresponding to an angular momentum of $60 \hbar$ we find that the width we calculate for the charge distribution of the projectile like fragment is $\Gamma_f = 1.30$. The numerically determined value [1], which includes the influence of damping, has the slightly larger value of 1.67.

The prospect that originally motivated this investigation was the hope that the width of the charge distribution in fission would provide a key to differentiating between two substantially different proposals for the path of a fissioning nucleus on its way from saddle to scission. In ref. [5] two different dynamical paths are determined for fission that both give the correct value for the asymptotic kinetic energy release. One trajectory is quite rapid and is only weakly damped by an ordinary hydrodynamic viscosity. The final velocity comes from the relative motion at scission and the Coulomb acceleration of the separating fragments. The other trajectory is based on the one-body damping mechanism and is much slower. The resulting scission shape is more compact and the relative motion is slower. However, the increased Coulomb acceleration after scission results in a final kinetic energy that is nearly the same as for the trajectory based on viscous damping.

The final width of the fragment charge distributions depends on the rate at which the neck between the two nuclei closes off at scission. In fig. 2 the inverse mass b (in the charge equilibration degree of freedom) is plotted as a function of time for the two different fission trajectories. (See the appendix for details of the calculation.) Since the rate at which b decreases is quite different, we expected that the predicted charge dispersions would also be quite different.

Unfortunately, the freeze-out of the charge dispersion occurs so late in the process (see fig. 3) that there is almost no difference between the two predictions. In fig. 3 the last part of fig. 2 is shown on an expanded scale. In the lower half of the figure the quantity C is plotted for the case where the fission trajectory was calculated using viscous damping. (The calculation of C was based on $k = 3.1$ MeV.) C attains the critical value of 1.108 at $t_c = -1.11$ dsec and the corresponding inverse mass is $b_c = 0.0338$ (MeV dsec²)⁻¹. This corresponds to a predicted charge dispersion of $\Gamma_f = 1.38$ which is somewhat smaller than the experimental value of 1.50. The quantity C for the case of the trajectory associated with one-body damping is not plotted since it lies nearly on top of the curve for viscous damping. Since the curves for b are so similar when freeze-out occurs, the predicted charge widths are nearly the same in the two cases. Consequently, we are forced to conclude that these considerations do not provide a means for choosing between the two fission trajectories.

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Appendix

The inertial mass associated with the hydrodynamical flow through a hyperbolic neck connecting the two halves of a fissioning nucleus was calculated in the Werner-Wheeler approximation. (See ref. [8].)

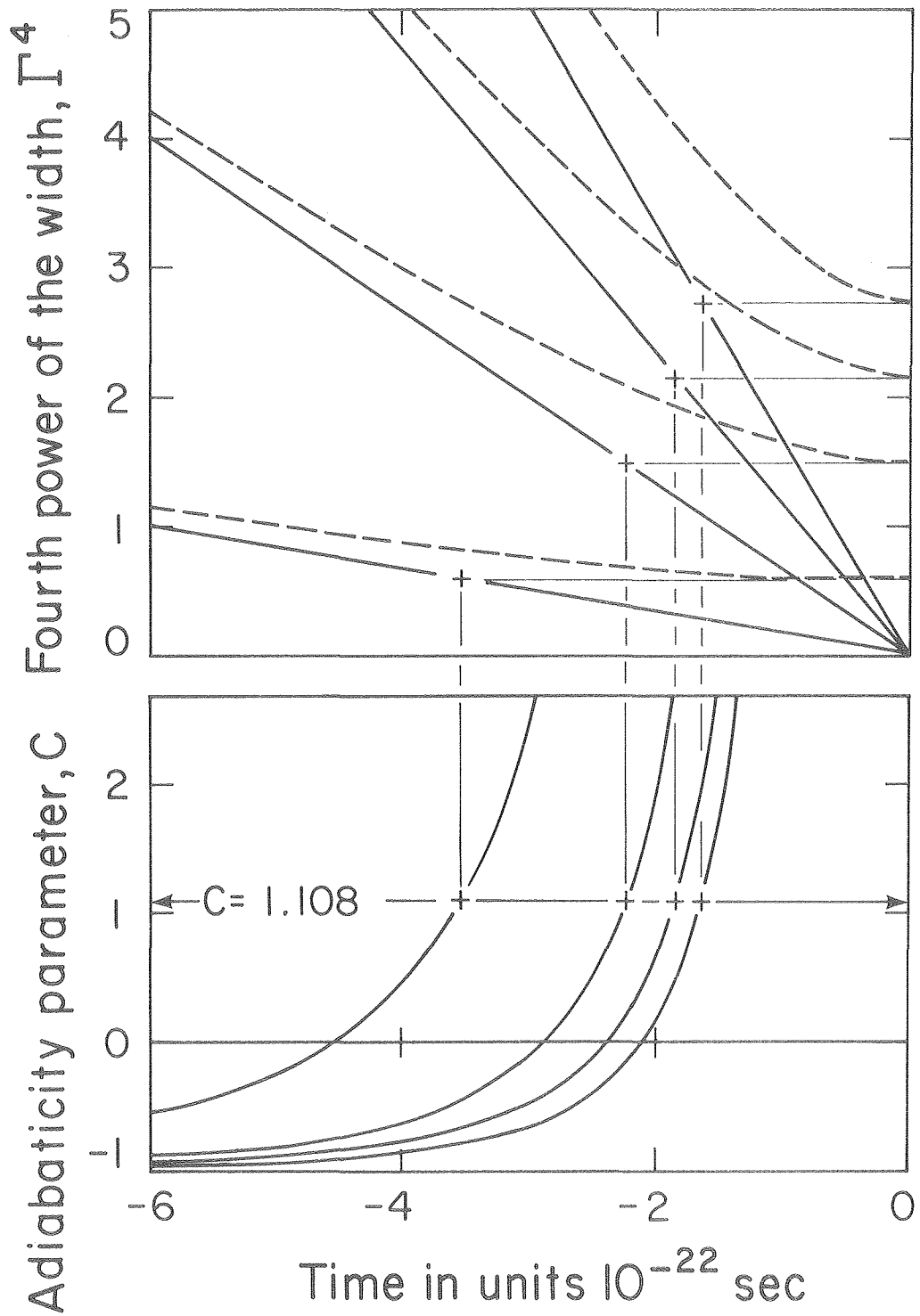
If the neck is described (in the cylindrical coordinates r and z) by the expression $r^2 = c^2 + \epsilon^2 z^2$ then the inertial mass per particle associated with the flow from $-z_2$ to $+z_2$ is given by the expression

$$M = \frac{m}{\rho} \left[\frac{1}{c\epsilon} \left(\frac{2}{\pi} \tan^{-1} z' \right) + \frac{\epsilon}{2\pi c} \left(\tan^{-1} z' - \frac{z'}{1+z'^2} \right) \right] ,$$

where m is the particle mass, ρ the particle number density and $z' = z_2 \epsilon / c$. If we drop the last two terms in the expression (which come from the radial part of the flow field) and combine the masses of the neutron and proton flows (for which $\rho \approx \rho_0/2$) we arrive at the approximate expression for b that was used for figs. 2 and 3,

$$b^{-1} = \frac{4m^*}{\rho_0} \frac{1}{c\varepsilon} \left(\frac{2}{\pi} \tan^{-1} z' \right) .$$

Here $m^* = 0.7m$ in accordance with earlier findings with regard to the GDR in spherical nuclei [9]. The quantity in parentheses takes the value unity when c (the neck radius) becomes small near scission. Consequently the value of b becomes independent of z_2 (the limit of integration) and is simply proportional to c as one would expect.



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Fig. 1

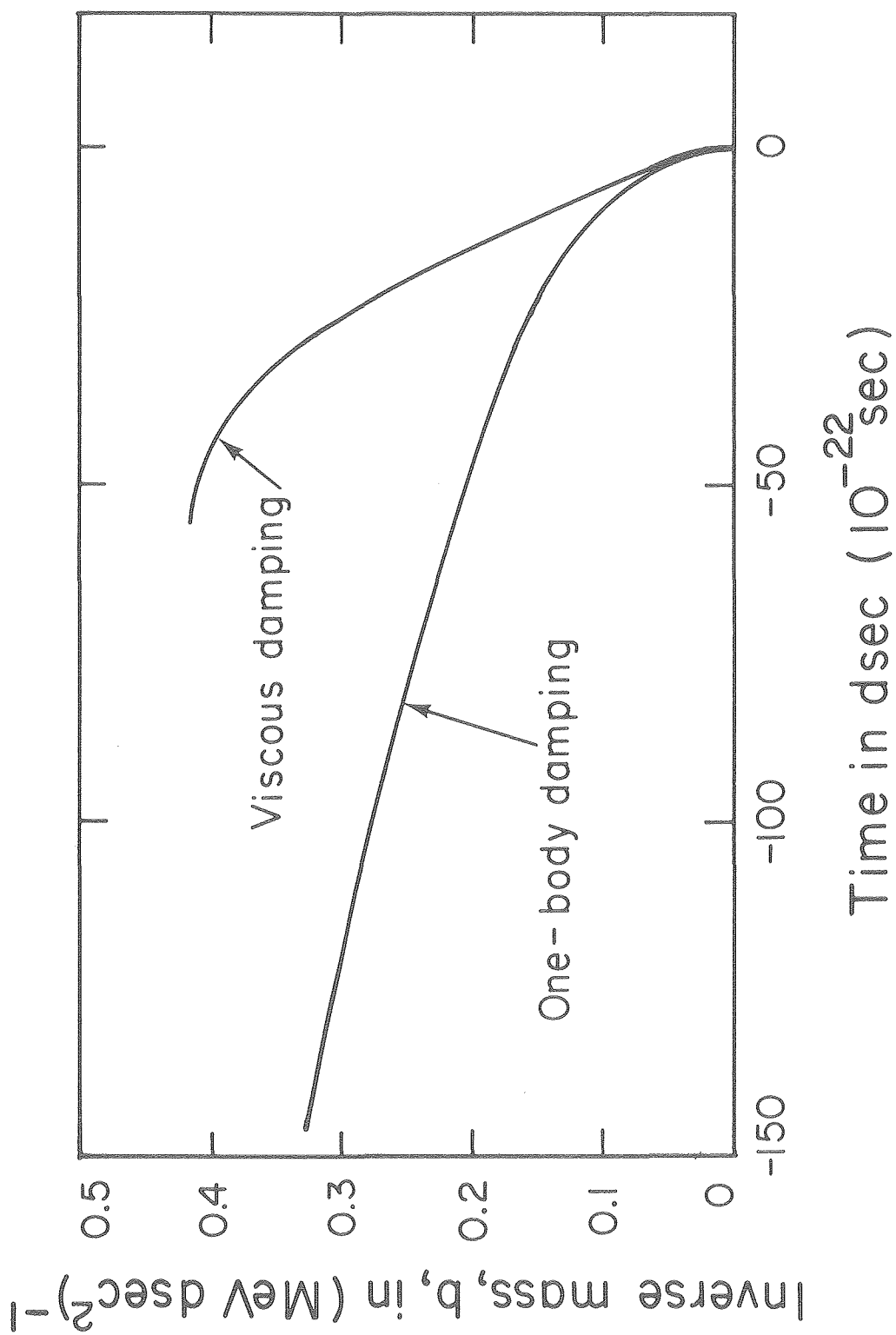
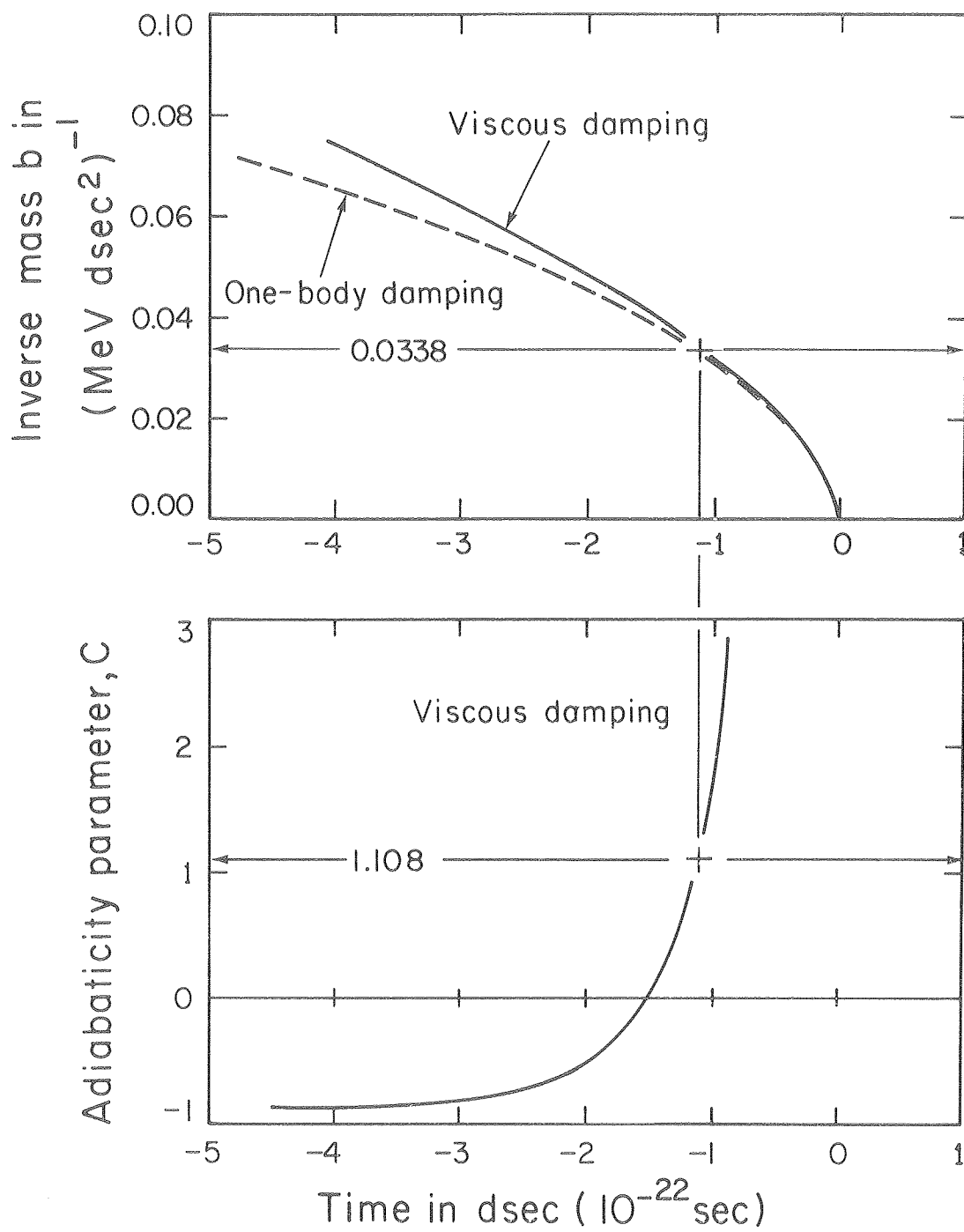


Fig. 2

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XBL 809-1914

Fig. 3